1 A company claims that a quarter of the bolts sent to them are faulty. To test this claim the number of faulty bolts in a random sample of 50 is recorded.

a Give two reasons why a binomial distribution may be a suitable model for the number of faulty bolts in the sample. **(2 marks)**

b Using a 5% significance level with the model of a binomial distribution, find the critical region for a 2-tail test of the hypothesis that the probability of a bolt being faulty is 0.25. The probability of rejection in each tail should be less than 0.025. **(4 marks)**

2 **a** Define the critical region of a test statistic. **(2 marks)**

A discrete random variable X has a binomial distribution B(30, p). A single observation is used to test H_0: $p = 0.3$ against H_1: $p > 0.3$

b Under H_0: X~B(30, 0.3), using a 1% level of significance find the critical region of this test. You should state, to 2 significant figures, the probability of rejection. **(3 marks)**

The value of the observation was found to be 15.

c Giving a reason, carefully state the outcome of the test. **(2 marks)**

3 A single observation x is to be taken from a binomial distribution B(20, p).

This observation is used to test H_0: $p = 0.3$ against H_1: $p \neq 0.3$

a Under H_0: X~B(20, 0.3), using a 5% level of significance, find the critical region for this test. You should state the probability of rejection in each tail, which should be less than 2.5 % **(3 marks)**

b State the actual significance level of this test. **(1 mark)**

The actual value of x obtained is 3.

c State a conclusion that can be drawn based on this value, giving a reason for your answer. **(2 marks)**

4 A single observation x is to be taken from a binomial distribution B(28, p).

This observation is used to test H_0: $p = 0.37$ against H_1: $p > 0.37$

a Use your calculator to find the critical region for this test. Use a 5% significance level and show your working clearly. **(3 marks)**

The actual value of x obtained is 17.

b State a conclusion that can be drawn based on this value, giving a reason for your answer. **(2 marks)**

5 Linda regularly takes a taxi to work five times a week. Over a long period of time she finds the taxi is late once a week on average. The taxi firm changes her driver and Linda thinks the taxi is late more often. In the first week with the new driver the taxi is late 3 times.

You may assume that the number of times the taxi is late in a week can be modelled with a binomial distribution.

a Test, at the 5% level of significance, whether or not there is evidence of an increase in the proportion of times the taxi is late. State your hypotheses clearly. **(6 marks)**

b One of the assumptions when modelling using a binomial distribution is that the probability of success p, in this case the probability the taxi is late, is constant throughout all the trials. Give two possible reasons why this assumption may not hold for this situation. **(2 marks)**

6 It is known from past records that 1 in 5 bowls produced in a pottery have minor defects. To monitor production a random sample of 25 bowls was taken and the number of such bowls with defects was recorded.

a Using a 5% level of significance, find critical regions for a 2-tail test of the hypothesis that 1 in 5 bowls have minor defects. The probability of rejection, in either tail, should be no more than 2.5%. **(6 marks)**

b State the actual significance level of the above test. **(1 mark)**

At a later date, a random sample of 20 bowls was taken and 2 of them were found to have minor defects.

c Test, at the 10% level of significance, whether or not there is evidence that the proportion of bowls with minor defects has decreased. State your hypotheses clearly. **(5 marks)**

7 Brad planted 25 seeds in his greenhouse. He has read in a gardening book that the probability of one of these seeds germinating is 0.25. Ten of Brad's seeds germinated. He claimed that the gardening book had underestimated this probability. Test, at the 5% level of significance, Brad's claim. State your hypotheses clearly. **(6 marks)**