AS and A level Mathematics Practice Paper – Coordinate geometry – Mark scheme

Question	Scheme	Marks
1	Mid-point of PQ is (4, 3)	B1
	PQ: $m = \frac{0-6}{9-(-1)}$, $\left(=-\frac{3}{5}\right)$	B1
	Gradient perpendicular to PQ $= -\frac{1}{m}$ $\left(=\frac{5}{3}\right)$	M1
	$y-3=\frac{5}{3}(x-4)$	M1
	$5x-3y-11=0$ or $3y-5x+11=0$ or multiples e.g. $10x-6y-22=0$	A1
		(5 marks)
2(a)	Method 1: $gradient = \frac{y_1-y_2}{x_1-x_2} = \frac{2-(-4)}{-1-7}, = -\frac{3}{4}$ Method 2: $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$, so $\frac{y-y_1}{6} = \frac{x-x_1}{-8}$	M1 A1
	$y-2=-\frac{3}{4}(x+1)$ or $y+4=-\frac{3}{4}(x-7)$ or $y = their\,'-\frac{3}{4}'x+c$	M1
	$\Rightarrow \pm(4y+3x-5)=0$	A1
	Method 3: Substitute $x = -1$, $y = 2$ and $x = 7$, $y = -4$ into $ax + by + c=0$	M1
	$-a+2b+c=0$ and $7a-4b+c=0$	A1
	Solve to obtain $a = 3$, $b = 4$ and $c = -5$ or multiple of these numbers	M1 A1
		(4)
2(b)	Attempts $gradient\ LM \times gradient\ MN = -1$ so $-\frac{3}{4}\times\frac{p+4}{16-7}=-1$ or $\frac{p+4}{16-7}=\frac{4}{3}$ Or $(y+4)=\frac{4}{3}(x-7)$ equation with $x =$ 16 substituted	M1
	$p+4=\frac{9\times 4}{3} \Rightarrow p = ...$, $p=8$ So $y =$, $y=8$	M1 A1
		(3)
2(c)	Either (y=) $p+6$ or $2+p+4$ Or use 2 perpendicular line equations through L and N and solve for y	M1
	(y =) 14	A1
		(2)
		(9 marks)

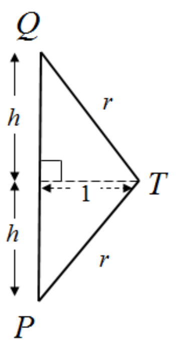
Question	Scheme	Marks
3(a)	Gradient of $l_1 = \frac{4}{5}$ oe	B1
	Point $P = (5, 6)$	B1
	$-\frac{5}{4} = \frac{y - "6"}{x - 5}$ or $y - "6" = -\frac{5}{4}(x - 5)$ or $"6" = -\frac{5}{4}(5) + c \Rightarrow c = ...$	M1
	$5x + 4y - 49 = 0$	A1
		(4)
3(b)	$y = 0 \Rightarrow 5x + 4(0) - 49 = 0 \Rightarrow x = ...$ **or** $y = 0 \Rightarrow 5(0) = 4x + 10 \Rightarrow x = ...$	M1
	$y = 0 \Rightarrow 5x + 4(0) - 49 = 0 \Rightarrow x = ...$ **and** $y = 0 \Rightarrow 5(0) = 4x + 10 \Rightarrow x = ...$	M1
	<u>Method 1:</u> $\frac{1}{2} ST \times "6"$ $\frac{1}{2} \times ('9.8' - '-2.5') \times '6' = ...$ **<u>Method 2:</u>** $\frac{1}{2} SP \times PT$ $\frac{1}{2} \times \sqrt{(5 - '-2.5')^2 + ('6')^2} \times \sqrt{('9.8' - 5)^2 + ('6')^2} = ...$ $\left(= \frac{1}{2} \times \frac{3\sqrt{41}}{2} \times \frac{6\sqrt{41}}{5}\right)$ **<u>Method 3:</u>** 2 Triangles $\frac{1}{2} \times (5 + '2.5') \times '6' + \frac{1}{2} \times ('9.8' - 5) \times '6' = ...$ **<u>Method 4:</u>** Shoelace method $\frac{1}{2}\begin{vmatrix} 5 & 9.8 & -2.5 & 5 \\ 6 & 0 & 0 & 6 \end{vmatrix} = \frac{1}{2}\left\|(0 + 0 - 15) - (58.8 + 0 + 0)\right\| = \frac{1}{2}\left\|-73.8\right\| = ...$ **<u>Method 5:</u>** Trapezium + 2 triangles $\frac{1}{2} \times ('2.5') \times '2' + \frac{1}{2}("2" + "6") \times 5 + \frac{1}{2} \times ("9.8" - 5') \times '6' = ...$	**dd**M1
	$= 36.9$	A1
		(4)
		(8 marks)

Question	Scheme	Marks
4(a)	(a) $2x+3y=26 \Rightarrow 3y=26\pm 2x$ and attempt to find m from $y=mx+c$	M1
	$\left(\Rightarrow y=\frac{26}{3}-\frac{2}{3}x\right)$ so gradient = $-\frac{2}{3}$	A1
	Gradient of perpendicular = $\frac{-1}{\text{their gradient}}$ $\left(=\frac{3}{2}\right)$	M1
	Line goes through (0,0) so $y=\frac{3}{2}x$	A1
		(4)
4(b)	(b) Solves their $y=\frac{3}{2}x$ with their $2x+3y=26$ to form equation in x or in y	M1
	Solves their equation in x or in y to obtain $x=$ **or** $y=$	dM1
	$x=4$ or any equivalent e.g. 156/39 or $y=6$ o.a.e	A1
	$B=\left(0,\frac{26}{3}\right)$ used or stated in (b)	B1
	Area = $\frac{1}{2}\times"4"\times\frac{"26"}{3}$	dM1
	$=\frac{52}{3}$ (oe with integer numerator and denominator)	A1
		(6)
		(10 marks)

Question	Scheme	Marks
5(a)	$L_1: 4y + 3 = 2x \Rightarrow y = \frac{1}{2}x - \frac{3}{4}$; $A(p, 4)$ lies on L_1. $\{p =\}\ 9\frac{1}{2}$ or $\frac{19}{2}$ or 9.5	 B1 **(1)**
5(b)	$\{4y + 3 = 2x\} \Rightarrow y = \frac{2x-3}{4} \Rightarrow m(L_1) = \frac{1}{2}$ or $\frac{2}{4}$ So $m(L_2) = -2$ $L_2:\ y - 4 = -2(x - 2)$ $L_2:\ 2x + y - 8 = 0$ or $L_2:\ 2x + 1y - 8 = 0$	M1 A1 B1ft M1 A1 **(5)**
5(c)	$\{L_1 = L_2 \Rightarrow\}\ 4(8 - 2x) + 3 = 2x$ or $-2x + 8 = \frac{1}{2}x - \frac{3}{4}$ $x = 3.5,\ y = 1$	M1 A1 A1 cso **(3)**
5(d)	$CD^2 = ("3.5" - 2)^2 + ("1" - 4)^2$ $CD = \sqrt{("3.5" - 2)^2 + ("1" - 4)^2}$ $= \sqrt{1.5^2 + 3^2} = 1.5\sqrt{1^2 + 2^2} = 1.5\sqrt{5}$ or $\frac{3}{2}\sqrt{5}$ **(*)**	"M1" A1 ft A1 **cso** **(3)**
5(e)	Area = triangle ABC + triangle ABE $= \frac{1}{2} \times \frac{3}{2}\sqrt{5} \times \sqrt{80} + \frac{1}{2} \times 3\sqrt{5} \times \sqrt{80}$ Finding the area of any triangle. $= \frac{3}{4}\sqrt{5} \times 4\sqrt{5} + \frac{3}{2}\sqrt{5} \times 4\sqrt{5}$ $= \frac{3}{4}(20) + \frac{3}{2}(20)$ $= 45$	 M1 B1 A1 **(3)**
		(15 marks)

Question	Scheme	Marks
6(a)	Gradient of l_1 is $\frac{7-2}{3-0}\left(=\frac{5}{3}\right)$	B1
	$m(l_2) = -1 \div their\ \frac{5}{3}$	M1
	$y-7 = "-\frac{3}{5}"(x-3)$ or $y = "-\frac{3}{5}"x + c,\ \ 7 = "-\frac{3}{5}"(3) + c \ \Rightarrow\ c = \frac{44}{5}$	M1A1ft
	$3x+5y-44=0$	A1
		(5)
6(b)	When $y=0$ $x=\frac{44}{3}$	M1 A1
		(2)
6(c)	Correct attempt at finding the area of any one of the triangles or one of the trapezia.	M1
	A correct numerical **expression** for the area of one **triangle** or one **trapezium** for **their coordinates**.	A1ft
	Combines the correct areas together correctly	dM1
	Correct numerical expression for the area of *ORQP*	A1
	Correct exact area e.g. $54\frac{1}{3}$, $\frac{163}{3}$, $\frac{326}{6}$, $54.\dot{3}$ or any exact equivalent	A1
		(5)
		(12 marks)
7	The equation of the circle is $(x+1)^2+(y-7)^2=(r^2)$	M1 A1
	The radius of the circle is $\sqrt{(-1)^2+7^2}=\sqrt{50}$ or $5\sqrt{2}$ or $r^2=50$	M1
	So $(x+1)^2+(y-7)^2=50$ or equivalent	A1
		(4 marks)

Question	Scheme	Marks
8(a)	$\{PQ =\}\ \sqrt{(7-10)^2+(8-13)^2}$ or $\sqrt{(10-7)^2+(13-8)^2}$	M1
	$\{PQ\} = \sqrt{34}$	A1
		(2)
8(b)	$(x-7)^2+(y-8)^2 = 34$ $\left(\text{or } \left(\sqrt{34}\right)^2\right)$	M1 A1 oe
		(2)
8(c)	$\{\text{Gradient of radius}\} = \frac{13-8}{10-7}$ or $\frac{5}{3}$	B1
	Gradient of tangent $= -\frac{1}{m}\left(= -\frac{3}{5}\right)$	M1
	$y - 13 = -\frac{3}{5}(x-10)$	M1
	$3x + 5y - 95 = 0$	A1
		(4)
		(8 marks)
9(a)	$x^2 + y^2 + 4x - 2y - 11 = 0$	
	$\left\{\underline{(x+2)^2 - 4} + \underline{\underline{(y-1)^2 - 1}} - 11 = 0\right\}$ — $(\pm 2, \pm 1)$, see notes.	M1
	Centre is $(-2, 1)$. — $(-2, 1)$.	A1 **cao**
		(2)
9(b)	$(x+2)^2 + (y-1)^2 = 11 + 1 + 4$ So $r = \sqrt{11+1+4} \Rightarrow r = 4$ — $r = \sqrt{11 \pm "1" \pm "4"}$	M1
	4 or $\sqrt{16}$ (Award A0 for ± 4).	A1
		(2)
9(c)	When $x = 0$, $y^2 - 2y - 11 = 0$ — Putting $x = 0$ in C or their C.	M1
	$y^2 - 2y - 11 = 0$ or $(y-1)^2 = 12$, etc	A1 aef
	$y = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-11)}}{2(1)}$ $\left\{= \frac{2 \pm \sqrt{48}}{2}\right\}$ — Attempt to use formula or a method of completing the square in order to find $y = ...$	M1
	So, $y = 1 \pm 2\sqrt{3}$ — $1 \pm 2\sqrt{3}$	A1 **cao cso**
		(4)
		(8 marks)

Question	Scheme	Marks
10(a)	$x^2 + y^2 - 10x + 6y + 30 = 0$	
	Uses any appropriate method to find the coordinates of the centre, e.g. achieves $\underline{(x \pm 5)^2} + \underline{\underline{(y \pm 3)^2}} = \ldots$. Accept (±5,±3) as indication of this.	M1
	Centre is $(5, -3)$.	A1
		(2)
10(b)	**Way 1**	
	Uses $\underline{(x \pm "5")^2 - "5^2"} + \underline{\underline{(y \pm "3")^2 - "3^2"}} + 30 = 0$ to give	
	$r = \sqrt{"25"+"9"-30}$ or $r^2 = "25"+"9"-30$ (not $30 - 25 - 9$)	M1
	$r = 2$	A1cao
	Way 2	
	Using $\sqrt{g^2 + f^2 - c}$ from $x^2 + y^2 + 2gx + 2fy + c = 0$ (Needs formula stated or correct working)	M1
	$r = 2$	A1
		(2)
10(c)	**Way 1**	
	Use $x = 4$ in *an* equation of circle and obtain equation in y only	M1
	e.g. $(4-5)^2 + (y+3)^2 = 4$ or $4^2 + y^2 - 10 \times 4 + 6y + 30 = 0$	
	Solve their quadratic in y and obtain **two** solutions for y	dM1
	e.g. $(y+3)^2 = 3$ or $y^2 + 6y + 6 = 0$ so $y = -3 \pm \sqrt{3}$	A1
	Way 2	
		
	Divide triangle *PTQ* and use Pythagoras with $"r"^2 - ("5" - 4)^2 = h^2$	M1
	Find h and evaluate $"-3" \pm h$ May recognise (1,√3, 2) triangle	dM1
	So $y = -3 \pm \sqrt{3}$	A1
		(3)
		(7 marks)

Question	Scheme	Marks
11(a)	**Mark (a) and (b) together** $OQ^2 = \left(6\sqrt{5}\right)^2 + 4^2$ or $OQ = \sqrt{\left(6\sqrt{5}\right)^2 + 4^2}$ $\{= 14\}$ $y_Q = \sqrt{14^2 - 11^2}$ $= \sqrt{75}$ or $5\sqrt{3}$	 M1 **d**M1 A1**cso** **(3)**
11(b)	$(x - 11)^2 + \left(y - 5\sqrt{3}\right)^2 = 16$	M1A1 **(2)**
		(5 marks)
12(a)	$A\left(\dfrac{-9+15}{2}, \dfrac{8-10}{2}\right) = A(3, -1)$	M1A1 **(2)**
12(b)	$(-9-3)^2 + (8+1)^2$ or $\sqrt{(-9-3)^2 + (8+1)^2}$ or $(15-3)^2 + (-10+1)^2$ or $\sqrt{(15-3)^2 + (-10+1)^2}$ Uses Pythagoras correctly in order to find the **radius**. Must clearly be identified as the **radius** and may be implied by their circle equation. Or $(15+9)^2 + (-10-8)^2$ or $\sqrt{(15+9)^2 + (-10-8)^2}$ Uses Pythagoras correctly in order to find the **diameter**. Must clearly be identified as the **diameter** and may be implied by their circle equation. This mark can be implied by just 30 clearly seen as the **diameter** or 15 clearly seen as the **radius** (may be seen or implied in their circle equation) **Allow this mark if there is a correct statement involving the radius or the diameter but <u>must be seen in (b)</u>** $(x-3)^2 + (y+1)^2 = 225$ $\left(\text{or } (15)^2\right)$ $(x-3)^2 + (y+1)^2 = 225$	M1 M1 A1 **(3)**
12(c)	Distance $= \sqrt{15^2 - 10^2}$ $\left\{= \sqrt{125}\right\} = 5\sqrt{5}$	M1 A1 **(2)**
12(d)	$\sin\left(\hat{ARQ}\right) = \dfrac{20}{30}$ or $\hat{ARQ} = 90 - \cos^{-1}\left(\dfrac{10}{15}\right)$ $\hat{ARQ} = 41.8103...$ awrt 41.8	M1 A1 **(2)**
		(9 marks)

AS and A level Mathematics Practice Paper – Coordinate geometry – Mark scheme

	Source paper	Question number	New spec references	Question description	New AOs
1	C1 2011	3	3.1	Straight lines	1.1b
2	C1 2017	8	3.1	Straight-line graph (perpendicular gradients)	1.1b, 2.1, 2.4 and 3.1a
3	C1 June 2014R	7	3.1	Equation of straight line and condition for perpendicularity	1.1b, 2.1, 2.2a, 3.1a
4	C1 2014	9	3.1	Coordinate geometry, perpendicularity	1.1b, 3.1a
5	C1 2012	9	3.1, 2.4	Straight lines, Indices and surds, Simultaneous equations	1.1a, 1.1b, 2.1, 2.2a, 3.1a
6	C1 2016	10	3.1	Lines, perpendicular	1.1b, 2.1, 2.4, ,3.1a
7	C2 Jan 2012	Q2	3.2	Circles	1.1b
8	C2 2016	3	3.1, 3.2	Circles	1.1b
9	C2 2011	Q4	2.3, 3.2	Circles	1.1b, 3.1a
10	C2 2017	5	3.2	Circles	1.1a, 1.1b, 3.1a
11	C2 2014	9	3.2	Circles	1.1b, 3.1a
12	C2 June 2014R	10	3.2	Circles	1.1b, 2.2a. 3.1a