## Contents

Lesson Page

1. Solving Linear Equations 1 ..... 3
2. Solving Quadratic Equations 1 ..... 7
3. Solving Quadratic Equations 2 ..... 11
4. Algebraic Fractions ..... 15
5. Iteration and Functions ..... 19
6. Simultaneous Equations 1 ..... 23
7. Simultaneous Equations 2 ..... 27
8. Inequalities ..... 31
9. Quadratic Inequalities ..... 35
10. Solving problems - Algebra ..... 39
11. Algebraic Proof ..... 43

## Solving Linear Equations 1

## Learning outcomes

1. Solve one and two step equations
2. Solve equations involving brackets
3. Solve equations involving fractions
4. Solve equations where the unknown appears on both sides of the equation

## Starter activity

Solve the 'Cross-Number'

| Across | Down |
| :--- | :--- |
| 1. $488+386$ | $1.55+31$ |
| 4.6 dozen | $2.58+29$ |
| $6.61 \times 11$ | $3.4385+796$ |
| $7.7 \times 29$ | $4.52 \times 14$ |
| $9.1086-268$ | $5.10 \times 2$ |
| $11.73-8$ | $8.2705+956$ |
| $13.221 \times 39$ | $10.6+13$ |
| $15.108-40$ | $12.1080-497$ |
| $16.16-3$ | $13.1092-281$ |
| $17.3526+987$ | $14.1263 \times 5$ |
| $19.5+6$ | $17.70-30$ |
| $20.764-160$ | $18.4389+1060$ |


| 1 | ${ }^{2}$ | 3 |  | $4^{4}$ | 5 |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 6 |  |  |  | 7 |  |  |  |

## Solving Linear Equations 1

## Activity 1

Solve the one and two step equations below
Section A: Section B:

1. $y+34=75=$
2. $2 m+6=36=$
3. $4 u=16=$
4. $6 j+20=2=$
5. $\frac{w}{5}=3=$
6. $x-6=-9=$
7. $p-17=9=$
8. $5-c=7=$
$5.4-e=0=$
9. $3 t-12=-24=$

## Activity 2

Solve the equations below, write your answer in the form $a=$ ?

1. $2(s+4)=6$
2. $-6=3(x+4)$
3. $48=4(3 a-3)$
4. $4(4 p-1)=-52$
5. $2(4 k+5)=26$

## Solving Linear Equations 1

## Activity 3

Solve the equations below, write your answer in the form $a=$ ?

1. $\frac{(r+4)}{4}=2$
2. $\frac{2(3+d)}{5}=8$
3. $6=\frac{5(2 a-2)}{10}$
4. $\frac{4(\mathbf{1}-\boldsymbol{k})}{2}=-12$
5. $-15=\frac{3(4-2 t)}{2}$
$\qquad$

## Activity 4

Solve the equations below, write your answer in the form $a=$ ?

1. $3 f+3=f+9=$
$2.7 s+23=12 s-2=$
$3.4-5 l=3 l+28=$
$4.2-6 z=-4 z+10=$
2. $-11 m-4=60-3 m=$

## Solving Linear Equations 1

## Plenary - What have I learnt today?

## Solving Quadratic Equations 1

## Learning outcomes

1. Factorise Quadratic Expressions
2. Solve Quadratic Equations by Factorisation
3. Solve Quadratic Equations using the Quadratic Formula

## Starter activity

Substitute the values into the following expressions

$$
\begin{array}{cc}
\text { If } h=10 \text { and } j=7, & \text { If } g=-13 \text { and } h=11, \\
\text { evaluate: } & \text { evaluate: } \\
h(10 h+2 j) & g h-8
\end{array}
$$

$$
\begin{gathered}
\text { If } p=0.6 \text { and } f=8, \\
\text { evaluate: } \\
9\left(p^{2}-2 f\right)
\end{gathered}
$$

If $y=-0.1$ and $a=8$ evaluate: $8 y a$

## Solving Quadratic Equations 1

## Activity 1

Factorise the following expressions

1. $x^{2}+11 x+28=$
2. $x^{2}-2 x-3=$
3. $x^{2}+15+8 x=$
4. $x^{2}+22+13 x=$
5. $x^{2}+12-7 x=$
6. $x^{2}-63-2 x=$
$4.15 x-16+x^{2}=$
$8.110-21 x+x^{2}=$

## Solving Quadratic Equations 1

## Activity 2

Factorise the following equations before solving
Section A:

1. $x^{2}-30+x=0$
$2.4 u-60+u^{2}=0$
2. $p^{2}-8 p-20=0$
3. $-40+3 y+y^{2}=0$

Section B:

1. $2 r^{2}+5 r+2=0$
2. $2 x^{2}-2-3 x=0$
$3.3 b^{2}+2+5 b=0$

## Solving Quadratic Equations 1

## Activity 3

Use the quadratic formula to solve the following equations

$$
x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}
$$

Section A:
$1.3 x^{2}-16 x=12$
2. $-37 h+10=-7 h^{2}$
3. $p^{2}-4 p+4=0$
4. $-7+6 y+y^{2}=0$

Extension:
5. $x^{2}-4 x+7=0$

Plenary - What have I learnt today?

## Solving Quadratic Equations 2

## Learning outcomes

1. Complete the square for quadratic expressions
2. Solve quadratic equations by 'completing the square'
3. Solve quadratic equations graphically

## Starter activity

Solve the following problems

1. The perimeter is 68 cm . Find s

$$
6 s+1
$$


2. The area is $35 \mathrm{~cm}^{2}$. Find r


## Solving Quadratic Equations 2

## Activity 1

Simplify the expressions by completing the square

1. $7-4 x+x^{2}$
2. $15-10 p+5 p^{2}$
$2.9-10 i+i^{2}$
$6.6 y+3 y^{2}-81$
3. $r^{2}+5 r-14$
$7.10 u^{2}+50 u-25$
4. $9 o+o^{2}-32$
5. $c-15+3 c^{2}$

## Activity 2

Complete the square and then solve the following equations
Leave your answers in surd form

1. $3 r^{2}-30+6 r=0$
2. $-8 x-4+2 x^{2}=0$
3. $u^{2}-7 u+50=0$
4. $-3 c=2 c^{2}-6$

## Solving Quadratic Equations 2

## Activity 3

Draw the graphs of the below equations to find the solutions

1. $x^{2}-3 x-18=0$

2. $2 x^{2}+11 x+15=0$


## Solving Quadratic Equations 2

## Activity 3 <br> 4. $3 x^{2}+x-4=0$



Plenary - What have I learnt today?

## Algebraic Fractions

## Learning outcomes

1. Simplify algebraic fractions
2. Add and subtract algebraic fractions
3. Multiply and divide algebraic fractions
4. Solve equations with algebraic fractions

## Starter activity

Simplify the expressions by factorisation:

1. $x^{2}-6 x+9$
2. $2 r+r^{2}-3$
$3.110-22 f+x^{2}$
$4.8 y^{2}-4 y$
$5.16 f r^{2}+2 f r+4 f^{2} r$

## Algebraic Fractions

## Activity 1

Express the algebraic fractions in their simplest form by cancelling out common factors

1. $\frac{16 i m^{2}}{22 i^{2} m r}$
2. $\frac{8 p^{3}+4 p^{2}}{(2 p+1)(p-7)}$
3. $\frac{x^{2}-7 x+12}{x^{2}-8 x+16}$
4. $\frac{4 t\left(9 c^{2}-30 c+25\right)}{2 t^{2}\left(5 c-25+6 c^{2}\right)}$

## Activity 2

Simplify the expressions below

1. $\frac{16}{7 p-4}+\frac{4}{p}$
2. $\frac{1-c}{5}+\frac{3 c-2}{c+4}$
3. $\frac{6-f}{4 f}-\frac{2 f}{2 f^{2}}$
4. $\frac{2}{x+4}-\frac{3}{x+2}$
5. $\frac{1-x}{(x+7)(x-4)}-\frac{3}{x+7}$
6. $\frac{3(x+3)}{4}-\frac{8}{2 x+6}$

## Algebraic Fractions

## Activity 3

Simplify the expressions below

1. $\frac{16}{8 p-4} \times \frac{2 p-1}{p^{2}}$
2. $\frac{14}{\lambda^{2}+6-5 d} \div \frac{7}{(d+5)(d-2)}$
3. $\frac{(1-c)(c+4)}{9 c-6} \times \frac{3 c-2}{c+4}$
4. $\frac{-x+x^{2}}{(x+2)} \div \frac{x^{3}-x^{2}}{(3 x+3)}$
5. $\frac{(f+7)(4+f)}{2(f-2)} \times \frac{16 f-32}{f^{2}-6 f-40}$
6. $\frac{x}{4+x^{2}} \div \frac{7 x^{2}-7 x}{2 x-1}$

## Activity 4

Solve the following equations by first simplifying:
$\frac{32}{x} \times \frac{1}{x}=32$
$\frac{x}{4} \times \frac{2(x+1)}{3 x}=4$

Extension:
$\frac{2 x}{8 x+4} \times \frac{4 x-2}{x}=14$

## Algebraic Fractions

## Plenary - What have I learnt today?

## Iteration and Functions

## Learning outcomes

1. Find approximate solutions to equations using iteration
2. Interpret expressions as functions with inputs and outputs
3. Find an inverse function
4. Find a composite function

## Starter activity

Solve the following equations:
a) $x^{2}-5 x+6=0$ by factorising
b) $x^{2}+8 x-3=0$ by using the quadratic formula
c) $x^{2}+8 x+10=0$ by completing the square

## Iteration and Functions

## Activity 1

Answer the questions

1. a) Show that $x^{3}-x-19=0$ can be written as $x=\sqrt[3]{x+19}$
b) Use the iterative formula $x_{n+1}=\sqrt[3]{x_{n}+19}$ to find $x_{4}$ to 2 decimal places. Start with $x_{0}=0$
2. a) Show that $\frac{x^{4}}{2}-3 x=0$ can be written in the form $x=\sqrt[4]{6 x}$
b) Use the iteration formula $x_{n+1}=\sqrt[4]{6 x_{n}}$ to find $x_{4}$ to 2 decimal places. Start with $x_{0}=4$

## Activity 2

1. If $f(x)=4-x$, calculate
a) $f(5)$
b) $f(0)$
c) $f(-4)$
2. If $f(x)=2 x^{2}-4$, calculate
a) $f(2)$
b) $f(0)$
c) $f(-1)$
3. If $f(x)=2 x+2$, solve
a) $f(x)=10$
b) $f(x)=26$
c) $f(x)=-2$
4. If $f(x)=x^{2}-x$, solve

$$
f(x)=30
$$

## Iteration and Functions

## Activity 3

1. Given $f(x)$ find $f^{-1}(x)$
a) $f(x)=x+4$
b) $f(x)=8 x$
C) $f(x)=\frac{x-3}{2}$
d) $f(x)=\frac{3 x-1}{4}$
2. Given $f(x)$ find $f^{-1}(x)$
a) $f(x)=\frac{x}{5}-8$
b) $f(x)=\sqrt{x}-8$
C) $f(x)=4 x^{2}+1$
d) $f(x)=\sqrt{18-2 x}$

## Activity 4

1. Given that: $f(x)=4 x-3$ and $g(x)=x^{2}$
a) Find $f g(2)$
b) Find $g f(4)$
c) Find $f f(5)$
d) Find $g g(6)$

## Iteration and Functions

Activity 4
2. Given that: $f(x)=6 x+5$ and $g(x)=4 x$
a) Find $f g(5)$
b) Find $g f(2)$
c) Find $f g(x)$

Plenary - What have I learnt today?

## Simultaneous Equations 1

## Learning outcomes

1. Solve linear simultaneous equations algebraically
2. Solve problems by forming and solving simultaneous equations
3. Solve simultaneous equations where one is linear and the other is non-linear algebraically

## Starter activity

Solve the equations using the quadratic formula
Leave your answers in surd form where appropriate

$$
x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}
$$

1. $x^{2}+7 x=20$
2. $7-x^{2}+10 x=0$
3. $3 x^{2}-4 x+1=0$

## Simultaneous Equations 1

## Activity 1

Solve using elimination or substitution

1. $3 y+2 x=54$
$2 y-2 x=16$
2. $6 p+3 h=45$
$-2 p+2 h=12$
3. $7 d-4 f=80$
$-80+4 f=3 d$
4. $4+y=8 g$
$-166-8 y=3 g$

## Simultaneous Equations 1

## Activity 2

Create equations and solve simultaneously

1. Emma buys 8 donuts and 8 muffins and in total they cost $£ 7.20$.

Phoebe buys 12 donuts and 8 muffins in the same shop. She spends $£ 8.80$. How much does a donut and a muffin cost?
2. Callum and Frank have $£ 26.80$ in total but Frank has $£ 6.80$ less than Callum. Can you set up a pair of simultaneous equations and solve to find out how much money each person has?

SCORE / 4

## Activity 3

Solve the equations below simultaneously to find all possible solutions

$$
\begin{gathered}
2 x-1=y \\
y=x^{2}-2 x+2
\end{gathered}
$$

$$
2 x^{2}+y^{2}=57
$$

$$
x+y=1
$$

$x-4 y+1=0$
$x^{2}-4 x y+y^{2}=13$

## GCSE MATHS

## Simultaneous Equations 1

## Plenary - What have I learnt today?

## GCSE MATHS

## Simultaneous Equations 2

## Learning outcomes

1. Solve equations simultaneously where one is non-linear, algebraically
2. Solve graphically, simultaneous equations where one is non-linear

## Starter activity

Complete the equations

1. $3 y+2 x=54$
$2 y-2 x=16$
2. $6 p+3 h=45$
$-2 p+2 h=12$
3. $7 d-4 f=80$
$-80+4 f=3 d$
4. $4+y=8 g$
$-166-8 y=3 g$

## Simultaneous Equations 2

## Activity 1

Solve the following simultaneous equations algebraically

1. $y+5=x$
$y=x^{2}+x-14$
2. $y-3=x^{2}-3 x$
$10 x-y=39$
3. $x+y=-7$
$y^{2}=29-x^{2}$
4. $8+4 y=-4 x$ $y=-x^{2}$

## Simultaneous Equations 2

## Activity 2

Solve the simultaneous equations graphically

1. $y^{2}=x$
$2 x=y+6$

|  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |

$$
\text { 2. } \begin{aligned}
& y+3=5 x \\
& x^{2}+x+1=y
\end{aligned}
$$

|  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |

## GCSE MATHS

## Simultaneous Equations 2

## Plenary - What have I learnt today?

## Inequalities

## Learning outcomes

1. Solve one and two step linear inequalities
2. Solve more complex linear inequalities
3. Represent solutions on a number line, using set notation and on a graph

## Starter activity

Solve the following one and two step equations

1. $x+5=8$
2. $3 p-5=4$
3. $\frac{(8-k)}{4}=5$
4. $2 d=6$
5. $\frac{f}{4}+6=1$
6. $5+\frac{1}{r}=1$
7. $\frac{e}{4}=6$
8. $3(h+2)=5$
9. $\frac{3}{t-6}=1$

## Inequalities

## Activity 1

Solve the following simple one and two step inequalities

1. $4-s \geq 5$
2. $2 f \leq 14$
3. $3 r-5<10$
4. $2(1-r) \geq 12$
5. $4 d-4<68$
6. $6 t-6>t$

## Inequalities

## Activity 2

Solve the following more complex linear inequalities including compound inequalities

1. $10<4 x-5 \leq 27$
2. $\frac{1}{2} \leq 9-2 d<4.5$
3. $-3 \leq \frac{e+3}{2}<-1$
4. $0.3>14-\frac{r}{4}>-3$

Activity 3
Express the following inequalities on a number line and then shade the region on a graph that satisfies them

1. Express the following inequalities on a number line from -5 to 5
(3 marks)
a) $\{x: x \geq-2\}$
b) $-4 \leq x<2$
c) $2 x-5 \geq 3$


## Inequalities

Activity 3
2. Shade the region on a graph that satisfies
(4 marks)

$$
\begin{aligned}
& 2 x-1<y \\
& 6-x \leq y \\
& y \geq 2
\end{aligned}
$$



SCORE _ / 7

Plenary - What have I learnt today?

## Quadratic Inequalities

## Learning outcomes

1. Recap solving linear inequalities
2. Solve quadratic inequalities
3. Represent quadratic inequalities on a number line, using set notation and graphically

## Starter activity

Solving quadratic equations

1. Solve the following quadratic equations by factorisation
a) $x^{2}-3 x-7=3$
b) $x^{2}-6 x-55=0$
c) $5 x^{2}+13 x=6$
2. Solve the following quadratic equation using the quadratic formula (leave your answers in exact form)
$3 x^{2}-16 x+5=3$

$$
x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}
$$

## Quadratic Inequalities

## Activity 1

Solve the following linear inequalities

1. $3 x-4 \geq 5-x$
2. $4<3 r-6 \leq 9$
3. $8 \leq 4 x-2<x+2$

## Activity 2

Solve the following quadratic inequalities

1. $3 x^{2}+2<149$
2. $x^{2}-9 \geq 16$
3. $2 x^{2} \geq 128$
4. $-5 x^{2}>-5$
5. $x^{2}+15>64$
6. $x^{2}-32>49$

## GCSE MATHS

## Quadratic Inequalities

## Activity 3

Solve the following quadratic equations, sketch the graph and express them on a number line

1. $x^{2}+5 x+6>0$

| 1 | 1 | 1 | $\mid$ | $\mid$ | 1 | 1 | 1 | $\mid$ | $\mid$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -5 | -4 | -3 | -2 | -1 | 0 | 1 | 2 | 3 | 4 |


|  |  |  |  |  |  |  |  |  |  | 5 |  |  |  |  |  | T |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  | 5 |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  | 4 |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  | - ${ }^{\text {a }}$ |  | 2 |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | - |
|  |  |  |  |  |  |  |  |  |  |  |  |  | 2 | 3 |  | 4 | 5 | 6 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  | 12 |  |  |  |  |  |  |  |  | - |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

2. $2 x^{2}-x-12 \leq x^{2}-2 x$


## Quadratic Inequalities

```
Activity }
3. \(x^{2}+5 x+1 \geq-3 x-6\)
```



Plenary - What have I learnt today?

## Solving Problems - Algebra

## Learning outcomes

1. Translate simple situations and procedures into algebraic expressions
2. Derive an equation and solve problems
3. Derive simultaneous equations, solve and interpret the solution

## Starter activity

Find the area of the following shapes


## Solving Problems - Algebra

## Activity 1

Form expressions for the perimeter and area of the following shapes

1. $8 p+7$

2. $2 t+3$

$3 t+4$

## Activity 2

Answer the following questions by deriving and solving equations

1. If the area of the square is equal to $49 \mathrm{~cm}^{2}$ find the length of the side $x$

2. Alan is $x$ years old. Jim is 7 years older than Alan and Geoff is twice the age of Jim. The sum of their ages is 49 . Find out the ages of the three
3. The sum of seven consecutive numbers is 133 .

Find the value of the seven numbers

## Solving Problems - Algebra

## Activity 3

1. John and Amy both went to the shops together.

John bought 8 tomatoes and 10 onions and spent $£ 9.80$ Amy bought 14 tomatoes and 5 onions and spent $£ 11.40$ Work out the value of 1 tomato and 1 onion.
2. The area of the rectangle is equal to the area of the square. Find the value of $x$ and therefore the area of both shapes.
$x+4$


## Solving Problems - Algebra

## Plenary - What have I learnt today?

## Algebraic Proof

## Learning outcomes

1. Understand algebraic proof and how it is defined mathematically
2. Use algebra to support and construct arguments
3. Apply algebraic proof to solve exam style questions

## Starter activity

The area of the larger rectangle has an area 4 times larger than the area of the smaller rectangle.
Calculate the dimensions of the smaller rectangle


## Algebraic Proof

## Activity 1

Answer the following problem by showing some examples and then proving that it is true for all values of $n$

1. Show that the sum of two consecutive numbers will equal an odd number with three examples
2. Prove that the sum of two consecutive numbers will always be an odd number

## Activity 2

Answer the questions below

1. Prove that the sum of the square of two consecutive odd numbers is always two more than a multiple of 8
2. Prove that $(8 n+2)^{2}-(8 n-3)^{2}$ is always a multiple of 5

## Algebraic Proof

## Activity 3

Answer the questions below

1. Prove that the sum of four consecutive whole numbers is always even, then show an example where the sum of four consecutive integers is not always divisible by 4.
2. The first five terms of an arithmetic sequence are: $\begin{array}{lllll}7 & 13 & 19 & 25 & 31\end{array}$
Prove that the difference between the squares of any two terms of the sequence is always a multiple of 24 .

## Algebraic Proof

Plenary - What have I learnt today?

## Notes

$$
\begin{aligned}
& \sum_{i=1}\left(x_{i}-x\right)=\sum_{i=1} x_{i}^{-1+x} \quad y= \\
& y=084^{x^{x}} \quad x+y=3 \\
& \sqrt{\frac{x}{y}}=c \hat{a} \frac{a^{2}+b^{2}=x}{} \\
& \begin{cases}A=\vec{a}(y-b x,-a)^{2}+ \\
& \sin A=\frac{1}{2} \\
, & 1\end{cases}
\end{aligned}
$$

